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Impact of Applying Stochastic Differential Equations on Accurately of Stock Prices Predicting with Comparison between Black-Scholes and Binomial Models

Hana Mohammed Soliman and Abdassalam B. H. Aldaikh
Department of Mathematics, Faculty of Science
Omar AL-Mukhtar University, El-Beida – Libya
Hanaalqtane123@gmail.com

Abstract

In financial markets, accurate option pricing is essential for investors and risk managers. Two widely used mathematical models for pricing European options are the Binomial Option Pricing Model and the Black-Scholes Model. This paper provides a comparative study between the two models by analyzing their assumptions, computational approaches, strengths, and limitations. While the binomial model offers a discrete-time framework and is particularly suitable for American-style options, the Black Scholes model provides a continuous-time analytical solution under specific assumptions.

A numerical example is used to illustrate how the two models produce similar results under certain conditions. Numerical results demonstrate that as the number of steps in the Binomial model increases, the results converge significantly toward the Black-Scholes model values, confirming the convergence between discrete and continuous models in financial mathematics.

Keywords: stochastic model, option pricing, black-scholes, binomial model.

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أثر تطبيق المعادلات التفاضلية العشوائية على دقة التنبؤ بأسعار الأسهم مع مقارنة بين نموذج بلاك–شولز والنموذج الثنائي

هناء محمد سليمان وعبد السلام بوحويش حمد الدايخ
قسم الرياضيات، كلية العلوم، جامعة عمر المختار، البيضاء–ليبيا
Hanaalqtane123@gmail.com

الملخص

في الأسواق المالية، يعد التسعير الدقيق لعقود الخيار أمرا أساسيا للمستثمرين ومديري المخاطر، ويعتبر كل من نموذج تسعير الخيارات الثنائي ونموذج بلاك– شولز من أكثر النماذج الرياضية استخداما في تسعير الخيارات الأوربية. تقدم هذه الورقة البحثية دراسة مقارنة بين النموذجين من خلال تحليل افتراضاتهما، وأساليبهما الحسابية، ونقاط القوة، والقيود الخاصة بكل منهما. ففي حين يوفر النموذج الثنائي إطارا زمنيا متقطعا مناسبا بشكل خاص للخيارات الأمريكية، فإن نموذج بلاك– شولز يقدم حلا تحليليا في الزمن المستمر في ظل افتراضات محددة.

وتم تصميم مثال عددي لتوضيح كيفية استخدام النموذجين وأعطى نتائج متشابهة في ظل ظروف معينة. وتظهر النتائج العددية إنه كلما ازداد عدد الخطوات في النموذج الثنائي، تقاربت النتائج بشكل ملحوظ نحو قيم نموذج بلاك– شولز، مما يؤكد التقارب بين النماذج ذات الزمن المتقطع وتلك ذات الزمن المستمر في الرياضيات المالية.

الكلمات المفتاحية: النموذج العشوائي، تسعير الخيارات، بلاك– شولز، النموذج الثنائي.

1. Introduction

Models based on SDE_s have been implemented with significant success across diverse research fields, such as biology, epidemiology, mechanics, economics and finance.

The foundational importance of these modeling concepts dates back to their emergence in 1905.[1] Option pricing constitutes an essential component of the options trading market. The theoretical exploration of this field dates back to 1900, when Bachelier

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introduced an early stochastic model for stock price dynamics based on the concept of random wandering in his work "Theory de la Speculation".[2]

Later, Black Scholes developed the Black-Scholes (BS) option pricing formula, which catalyzed extensive academic interest in the subject.[3]

During the same year, Robert Merton refined this framework, leading to the establishment of the Black-Scholes-Merton (BSM) Model.[4]

The Black-Scholes-Merton model, frequently referred to as the Black-Scholes model, serves a mathematical framework for financial derivative markets used to derive the Black-Scholes formula. This formula provides estimates for the pricing of European call and put options and represents the first mathematically rigorous approach to be widely adopted for option valuation. The model is credited by many for the substantial growth in options trading and continues to exert a profound influence on contemporary financial pricing.

Before its introduction, traders lacked a uniform mathematical methodology for valuing options. However, empirical studies have demonstrated that the price estimates generated by this formula align closely with observed market prices.[5]. Following the introduction of the Black-Scholes formula for European call options in 1973, numerous economic models have been developed and documented. Notable examples include the Heston model, the jump-diffusion model, the constant elasticity of variance (CEV) model, and the Cox-Ingersoll-Ross (CIR) model. [6]

A highly effective and widely utilized method for option valuation involves the construction of a binomial tree. This diagrammatic representation illustrates the various potential trajectories a stock price may follow throughout the duration of an option. This approach is founded on the fundamental assumption that the stock price performs a random walk; specifically, within each time interval, there is a defined probability of the price increasing or decreasing by a specified percentage. In the limit, as the time steps approach zero, this model aligns with the Black-

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Scholes-Merton framework. Furthermore, the European option price derived from the binomial tree converges to the Black-Scholes-Merton valuation as the time increments diminish. [7]. This paper begins by introducing the concept of financial options and discussing their main classifications, including European and American options, as well as call and put options. It then presents the theoretical foundations of the Black–Scholes model, including its underlying assumptions and pricing formula, followed by a numerical example illustrating its application in option valuation. The study subsequently introduces the Binomial Option Pricing Model, highlighting its fundamental assumptions and pricing methodology. To demonstrate the implementation of this model, numerical examples based on both one-step and two-step binomial trees are provided. These examples illustrate the step-by-step valuation process and emphasize the flexibility of the binomial approach in modeling the evolution of the underlying asset price.

Finally, the paper presents a comparative analysis of the Black–Scholes and Binomial models.

2. Some Basic Concepts:

2.1 Options

Definition. An option is an accord among a purchaser and a vendor that grants one person the ability, but not the responsibility, to purchase or sell a stock until a predetermined time, while the other person is required to carry out the terms of the agreement upon request. [8]

There are two primary categories of options:

- European-style options: Are only exercisable at the date of maturity.
- American-style option: Can be exercised at any point prior to their expiration date of validity, unlike European options.[8]

The two primary kinds of options are as follows:

1- A purchase option: Grants its holder the ability to purchase a stock S_0 at a price known as the exercise price or strike price K up

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until a predetermined period of time known as the options maturation or expiry date T.

The option is meaningless and useless if it is not exercised in the date of expiry, meaning that the holder chooses not to purchase the stock that is underlying.

The option is utilized and the owner earns $S_0 - K$ by buying at K and then selling in the marketplace at S_0if the stock value is higher than K at T. The option is meaningless to the owner if the stock value is lower than K.

For the owner of a European call at T, the payoff is

$$\max[S_0 - K, 0] = (S_0 - K)^+ \tag{1}$$

where S_0 current price, and K is strike price.[8]

2- A sale option: Grants its holder the ability to sell a stock for a strike price through the option's maturity date.

The payoff of a European put option is given by a similar logic as

$$\max[K - S_0, 0] = (K - S_0)^+ \tag{2}$$

2.2. Option Price (Premium):

Definition. The premium is the market price of the option, which should not be combined with the value of the shares that is being traded. The premium is influenced by several variables: The current share price S, the strike price K, the amount of time left until the end of the period T, and the share's volatility σ are some of the variables that affect the premium. [8]

3. BSM Presumptions:

The BSM presumptions are:[7]

1. The fundamental stock's value follows a geometric Brownian motion with constants μ and σ.
2. It is acceptable for one to short securities and utilize the revenue in fully.
3. No exist any duties or transactional expenses. Every security can be divided correctly.
4. Throughout a duration of the derivatives, not exist profits.
5. There are no riskless arbitrage opportunities.

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6. Trade in securities is ongoing.
7. During every maturity, the risk-free rate of interest, or r, remains fixed.

4. Black-Scholes Closed Form:

The European call options cost in the BSM is

$$C(0,S_0) = S_0 N\big(d_1(T,S_0)\big) - Ke^{-rT} N\big(d_2(T,S_0)\big), \tag{3}$$

where d_1 and d_2 are defined as follows:

$$d_1(t,S) = \frac{ln\left(\frac{S}{k}\right) + \left(r + \frac{\sigma^2}{2}\right)t}{\sigma\sqrt{t}},$$
$$d_2(t,S) = d_1 - \sigma\sqrt{t}.$$

It should be noted that the premium depends on the volatility σ rather than the mean amount of returns μ, which is typically undefined.In the BSM, the European put option price is computed in exactly at the same manner.

$$P(0,S_0) = Ke^{-rT} N(-d_2(T,S_0) - S_0 N\big(-d_1(T,S_0)\big) \tag{4}$$

4.1 Example:

Consider a stock with a current price of $100. A European option on this stock expires in six months ($T = 0.5$), the exercising value is $100, the interest rate without risk is 5% annually, and the likelihood of volatility is 20% annually. That is to say:

$$S_0 = 100, \quad K = 100, \quad r = 0.05, \quad \sigma = 0.2, \quad T = 0.5,$$
$$d_1 = \frac{ln(100/100) + \left(0.05 + \frac{0.2^2}{2}\right)0.5}{0.2\sqrt{0.5}} = 0.2475,$$
$$d_2 = 0.2475 - 0.2\sqrt{0.5} = 0.1061.$$

Therefore, if the option is a European call, the worth C isdetermined as

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$$C = 100N(0.2475) - 100e^{-0.05\times 0.5}N(0.1061).$$

If the option is a European put, the worth P is determined as

$$P = 100e^{-0.05\times 0.5}N(-0.1061) - 100N(-0.2475).$$

By using standard normal distribution tables or a financial calculator, the values of the standard normal distribution function can be determined as follows:

$$N(0.2475) = 0.5977\,, N(-0.2475) = 0.4023,$$
$$N(0.1061) = 0.5423\ \ , N(-0.1061) = 0.4578.$$

in order for

$$C = \$6.88, \qquad P = \$4.42\,.$$

5. The Binomial Model (BM)

Cox et al. (1979) proposed the binomial option pricing model, which depends on a discrete-time framework that supports two possible markets outcomes $u(up), d\ (down)$ at any given period.[9]
Designing a binomial tree is a common and widely employed way of pricing an option. The graph below illustrates all of the pathways that the shares price could take throughout the course of an option. The fundamental premise is that the shares value exhibits a fluctuating pattern. It has a specific likelihood of rising uphill by some percentage and a definite probability of moving downhill by an identified percentage in every period.[7]

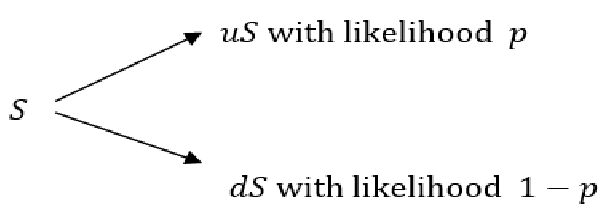

Fig. 1: One-step binomial tree for asset price evolution.

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Originally put forward by Cox, Ross, and Rubinstein 1979, the binomial tree methodology ranks as one of the more utilized techniques for determining pricing in diffusion models. [9]

5.1 The Binomial Model's Presumptions:

1. In discrete intervals, the asset price movement follows a multiplicative binomial pattern.
2. There are two conceivable outcomes the share's return during each phase:
$u-1$, with likelihood p, or $d-1$, with likelihood $1-p$. The value of the shares at the finish of the term thus will be uS or dS if the present value of the shares is S.
3. The risk-free interest rate is positive and steady throughout the period.
4. Exist absolutely no charges for transactions fees, charges. As a result, it is possible to sell any kind of security short while retaining all of the profit.[9]

5.2 A Single-Step BM Example

Given an option on a shares having a present value of C and an initial price of $S_0 = \$100$ Provided that the option has a duration of $T = 6\ months$, the price of the share may go rise from S_0 to a higher level, $S_0u = \$115.19$, and $u > 1$, or fall from S_0 to a lower level, $S_0d = \$86.81$, and $d < 1$.

The objective is to evaluate a European call option to buy the shares for $K = \$100$ in 6 months.

At the end of this period, the option is going to have just one of two values:

If the stock price rises to \$115.19, the payoff is $C_u = \$15.19$.

If the stock price falls to \$86.81, the payoff is $C_d = \$0$. Fig.2 depicts the case.

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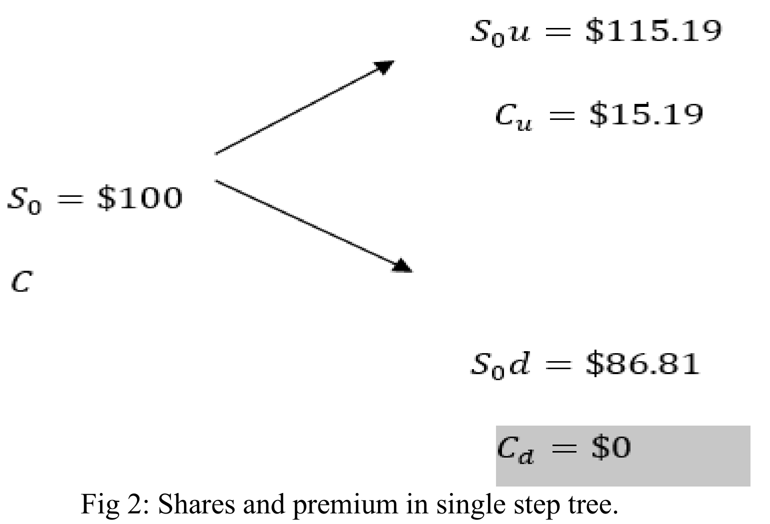

Fig 2: Shares and premium in single step tree.

The formula for the option price is derived as:

$$C = e^{-rT}[pC_u + (1-p)C_d] \ , \qquad (5)$$

where p represents the risk-neutral probability, defined as:

$$p = \frac{e^{rT} - d}{u - d} \ . \qquad (6)$$

When shares value fluctuations are provided by a single-step binomial tree, an option can be valued utilizing equations (5) and (6).

In the prior mentioned numerical example (refer to Fig 2),

$$u = 1.1519, \qquad d = 0.8681, \qquad r = 0.05,$$
$$T = 0.5, C_u = 15.19\, , C_d = 0.$$

Equation (6) yields:

$$p = \frac{e^{0.05\times 6/12} - 0.8681}{1.1519 - 0.8681} = 0.538,$$
$$1 - p = 0.462$$

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Thus, the option price is obtained from Equation (5) as follows

$$C = e^{-0.05\times 0.5}(0.5539 \times 15.19 + 0.4461 \times 0) = 8.21$$

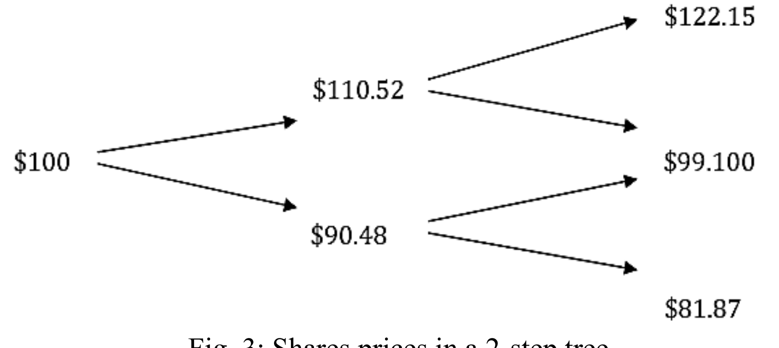
This result is consistent with the previously determined answer.[7]

5.3 Two-Step Binomial Tree:

The scope of the study is further extended to encompass a two-step binomial tree, like the one depicted in Fig.3

In this instance, the shares value begins at $100 and could rise or fall by 10% in each of the two-time phases, and $r = 5\%$ annually, and each step lasts 3months.

A 6-month option is examined option with an exercise price $100. Calculating the premium at the tree's starting point is the objective of the analysis. This is achieved through by the iterative application the previously mentioned concepts.



Fig, 3: Shares prices in a 2-step tree.

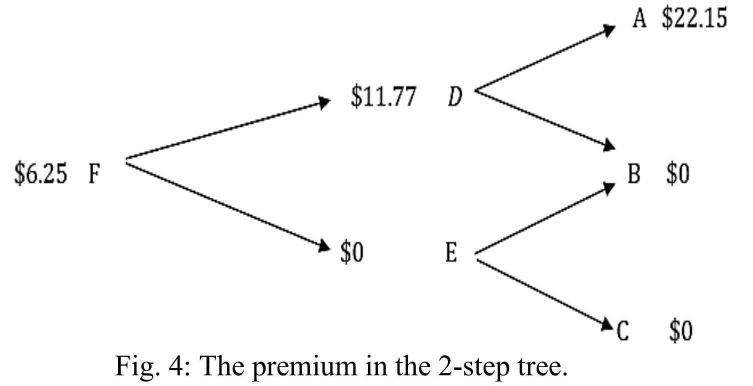


Fig. 4: The premium in the 2-step tree.

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Fig. 4, demonstrates the premium in each point, the premium in the terminal points of the branch can be readily computed.
In points B and C, the premium is out-of-the-money and has no worth, in point A, $C_{uu} = \$22.15$ and the premium is $122.15 - 100 = 22.15$.
Because point E goes either to point B or C, the two of which have 0 premium, point E has a 0 premium.
Utilizing $u = 1.1052,\ d = 0.9048,\ r = 0.05, T = 0.5 and p = 0.538$
and the premium worth in point Dis provided by Equation (5) with $r = 0.05\ and p = 0.538$:

$$e^{-0.05\times\frac{3}{12}}(0.538 \times 22.15 + 0.462 \times 0) = \$11.77$$

Similarly, the premium in point F is determined by focusing on the first phase of the tree. Given the values in pointsE (0) and D(11.77), the worth in point F is:

$$e^{-0.05\times\frac{3}{12}}(0.538 \times 11.77 + 0.462 \times 0) = \$6.25$$

By considering the general scenario in Fig 5, this methodology can be applied to any 2-interval case.
At first, the shares value is S_0. It potentially advances upwards to u times its value at the start of the time period or lowers to d times this worth through every phase of time.
The tree displays the code for the premium worth. (For instance, the premium worth is C_{uu} following two upward moves.) It is assumed that the time period is Δt years long and the risk-free interest rate is r. because a time period's length is Δt instead of *T*.

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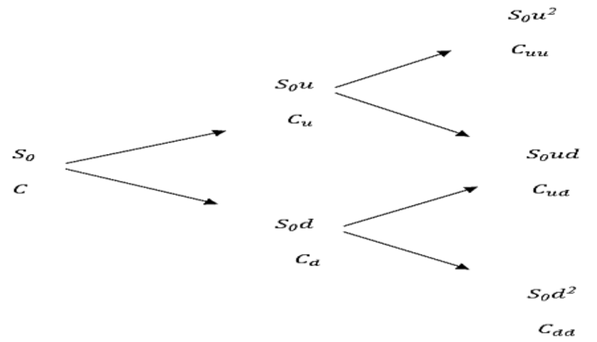

Fig 5: Shares and premium in general 2 step tree.

Equation (5) and (6) are expressed for a single step as:

$$C = e^{-r\Delta t}[pC_u + (1-p)C_d] \tag{7}$$

$$p = \frac{e^{r\Delta t} - d}{u - d} \tag{8}$$

Recursive application of Equation (7) yields

$$C_u = e^{-r\Delta t}[pC_{uu} + (1-p)C_{ud}], \tag{9}$$

$$C_d = e^{-r\Delta t}[pC_{ud} + (1-p)C_{dd}], \tag{10}$$

$$C = e^{-r\Delta t}[pC_u + (1-p)C_d]. \tag{11}$$

Equation (9) and (10) can be substituted into (11) to obtain

$$C = e^{-2r\Delta t}[p^2C_{uu} + 2p(1-p)C_{ud} + (1-p)^2C_{dd}]. \tag{12}$$

6. Comparison Between (BM) And (BSM):

6.1 Illustrative Example

To illustrate the difference between the BSM and the BM, take the previous numerical example for pricing a European call option. This section provides a comparison between the results of the BSM and BM. The BM is extended to three-time steps ($n = 3$) to analyze

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the convergence toward the BSM outcomes. The parameters are defined as follows:

$S_0 = \$100, \quad K = \$100, \quad T = 0.5, \quad \sigma = 20\%, \quad r = 5\%$

Step 1: compute model parameters

$$\Delta t = \frac{T}{n} = \frac{0.5}{3} \approx 0.1667$$
$$u = e^{\sigma\sqrt{\Delta t}} = e^{0.2\sqrt{0.1667}} \approx 1.0851$$
$$d = e^{-\sigma\sqrt{\Delta t}} = e^{-0.2\sqrt{0.1667}} \approx 0.9215$$
$$R = e^{r\Delta t} = e^{0.05\times 0.1667} \approx 1.0083$$
$$p = \frac{e^{r\Delta t} - d}{u - d} = \frac{1.0083 - 0.9215}{1.0851 - 0.9215} \approx 0.5306$$

Step2:Build the stock price tree

The calculation shows stock prices at each node up to 3 steps, as illustrated in Fig. 6.

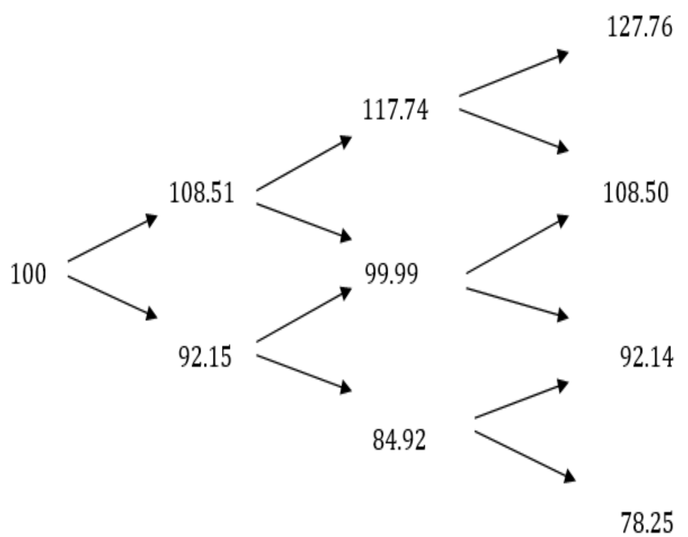

Fig. 6: Shares prices in a 3-step tree.

Step 3:Backward induction (see Fig 7)

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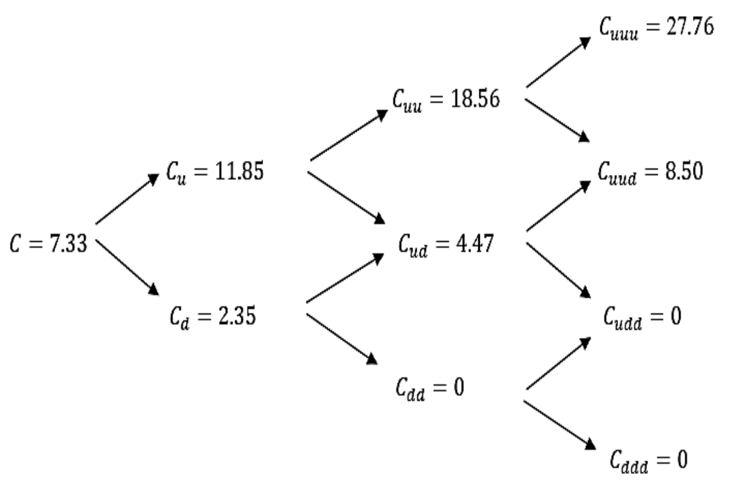

Fig. 7: The premium in the 3-step tree.

6.2 DerivaGem Software

DerivaGem Software is a professional tool designed for binomial tree applications. It can display trees with up to 10 steps, while calculations can be performed for up to 500 steps. In this study, DerivaGem is utilized for 3, 7 and 10 steps to demonstrate the convergence between manual calculations and the Black-Schools Model. It is observed that as the number of steps increases, the option price approaches the value derived from the Black-Scholes Model.

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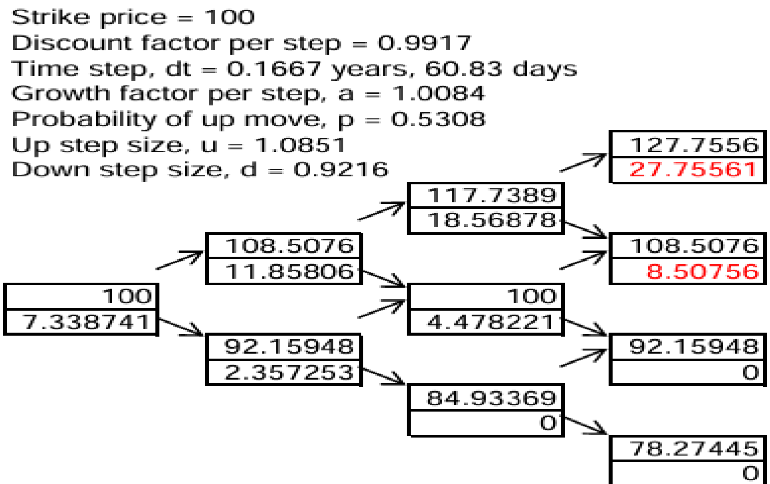

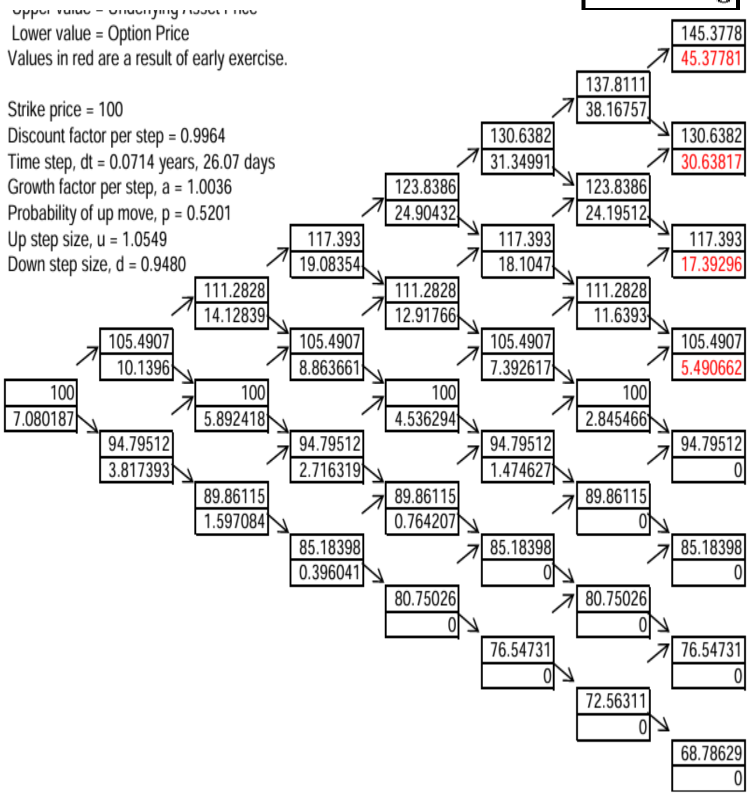

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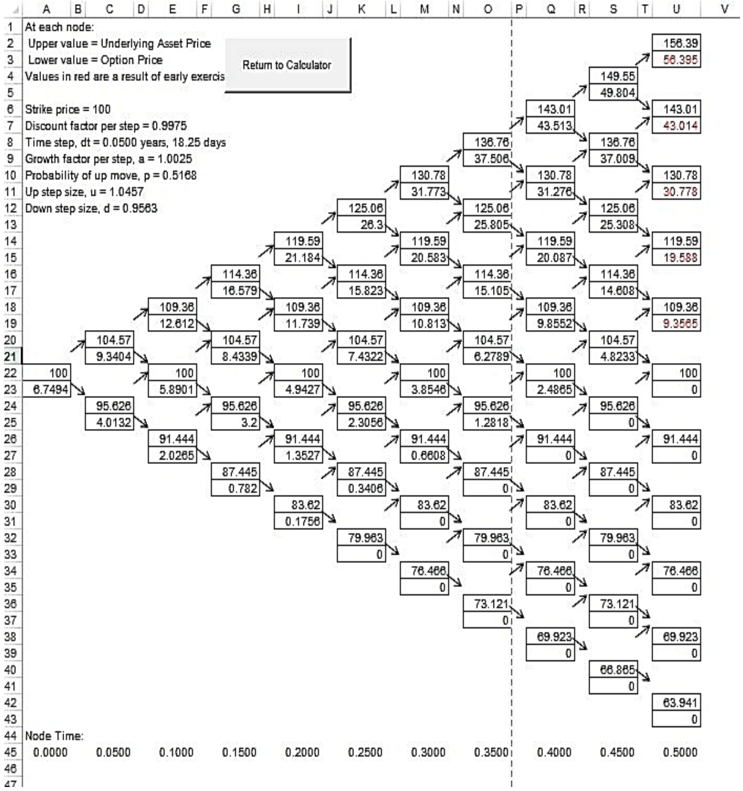
The binomial tree is further extended to $n = 7$ steps, as illustrated in the figure. The computed option price at the initial node is 7.080, which shows a closer approximation to the Black-Scholes price compared to the 3-step model. This refinement demonstrates the numerical convergence of the Binomial Model as the time interval Δt approaches zero.



The binomial model is further refined by increasing the number of steps to $n = 10$.

The resulting tree yields an option price of 6.7494. This progression illustrates the convergence property of the binomial model; as n increases, the discrete-time price provides a more accurate approximation of the continuous-time Black-Scholes value.

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The following table summarizes the convergence of the model across different steps.

Table1: Comparison of option prices between BSM and BM at different steps.

Model	Option price
Black scholes	\$6.88
Binomial (1 step)	\$8.21
Binomial (3 steps)	\$7.33
Binomial (7 steps)	\$7.08
Binomial (10 steps)	\$6.75

The results presented in Table (1) illustrate the convergence property of the BM toward the BSM. It is evident that as the number of time steps (n) increases, the estimated option price derived from the BM approaches the theoretical value of the BSM. Specifically, the price moves from $8.21\, atn = 1$ to $6.75\, atn = 10$, thereby reducing the approximation error. This numerical evidence supports the theoretical transition from discrete-time modeling to continuous-time modeling as $\Delta t \to 0$.

7. Conclusion

The comparison between the Black-Scholes and Binomial models has proven that financial mathematics is capable of providing accurate estimates for asset pricing despite market randomness.

The numerical findings demonstrated that the Binomial model, despite its structural simplicity compared to Black-Scholes, possesses high flexibility in simulation. Furthermore, increasing the number of time steps necessarily leads to the disappearance of the gap between continuous and discrete results.

Thus, this paper provides a theoretical and practical framework that assists researchers and financial analysts in selecting the most appropriate model based on the nature of the available data and the required precision.

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